NATURAL VIBRATIONS OF VISCOELASTIC THREE-LAYER SHALLOW SPHERICAL SHELLS

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Annotation

The development of various fields of engineering and technology is inextricably linked with the creation of new polymeric and composite materials with predetermined various physical and mechanical properties. The paper considers linear natural vibrations of a viscoelastic three-layer shallow spherical shell with the most common types of boundary conditions (sliding pinching, free support, etc.). The problem is reduced to the consideration of a single-layer spherical shell. The aim of the work is to formulate and solve the problem of natural oscillations of viscoelastic multilayer spherical shells. Based on the methods of differential equations in partial derivatives, a frequency equation with complex output parameters is obtained. The problem of natural oscillations of viscoelastic multilayer spherical shells is posed and solved. An equation of resonant frequencies is obtained, which makes it possible to implement the construction of eigenmodes and numerical results**.**

Keywords: spherical shell, sliding pinch, oscillation, viscoelastic body.

1. Introduction

The development of various fields of technology and industry is inextricably linked with the creation of new polymer and composite materials with various predetermined physical and mechanical properties [1,2]. The appearance of such materials is widely used in new technology and contributes to the creation of new structures operating under non-standard conditions, such as random non-stationary effects, pressure of a moving load, shock waves and currents, and seismic effects [3,4]. Therefore, taking into account the dependence of the mechanical properties of deformable media on the types of loading becomes more and more necessary, ignoring which can lead to loss of bearing capacity and destruction of the structure during operation [5,6]. As a result, there is a great need to create refined mathematical models, development of new methods solutions and calculation algorithms for theoretical research behavior of materials during operation under working conditions. The creation of software systems makes it possible to optimize and control the operation of structures during intensive non-stationary external loads [7,8]. In the absence of external loads, natural oscillations are considered [9].

2. Methods

2.1. Statements of the problem and basic relations

When obtaining a three-layer shell, it is assumed that the shells are considered asymmetric in thickness, the materials of the layers are isotropic, and their Poisson ratios are equal to each other.

$$\widetilde{D}_{1,2,3}\phi(t) = D_{01,02,03}\left[\phi(t) - \int_0^t R_{d1,2,3}(t-\tau)\phi(t)d\tau\right]; \widetilde{B}\phi(t) = B_{01}\left[\phi(t) - \int_0^t R_b(t-\tau)\phi(t)d\tau\right]. (1)$$

We take the integral terms in (1) small, then the integrands $\phi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- slowly changing function of time, ω_R- real constant. Next, applying the freezing procedure [10], we replace the integral relations with approximate ones of the form

$$\bar{D}_\kappa[f] = D_{0\kappa}\left[1 - \int_0^\infty R_{d\kappa}(\tau)\cos\omega_R\tau d\tau - i\int_0^\infty R_{d\kappa}(\tau)\sin\omega_R\tau d\tau\right]f,$$

$$\bar{B}_\kappa[f] = B_{0\kappa}\left[1 - \int_0^\infty R_{b\kappa}(\tau)\cos\omega_R\tau d\tau - i\int_0^\infty R_{b\kappa}(\tau)\sin\omega_R\tau d\tau\right]f, \quad (2)$$

where $f(t)$– arbitrary function of time. Then the integro-differential equations take the following form [11]:

$$\nabla^2\nabla^2\psi - \frac{\bar{B}_1(1-\nu^2)}{R}\nabla^2 w = 0,$$

$$\nabla^2[(\bar{D}\nabla^2 - 2\bar{G}h)\varphi - \bar{D}_3\nabla^2 w] = 0, \nabla^2\left[\frac{\bar{D}_1(1-\nu^2)}{2}\nabla^2 - 2\bar{G}h\right]\chi = 0, \quad (3)$$

$$\bar{D}_2\nabla^2\nabla^2 w - \bar{D}_3\nabla^2\nabla^2\varphi + \frac{1}{R}\nabla^2\psi + T\nabla^2 w - \Lambda^2 w = 0.$$

where ψ, w - complex functions of forces and deflection; φ, χ - complex functions of transverse shear; $\Lambda^2 = m\omega^2$; $\omega = \omega_R + i\omega_I$- complex frequency, m- mass of all layers per unit area, **w** - shell displacement vector, $\lambda_{0к}, \mu_{0к}$ - instantaneous moduli of elasticity, $c_{20} = [kE_0/2\rho_0(1+\nu_0)]^{1/2}$. The three-parameter relaxation kernel of Koltunov-Rzhanitsyn was used in the calculations: $R_k(t) = A_k e^{-\beta_k t}/t^{1-\alpha_k}$.

Under boundary conditions $\chi = \frac{\partial\chi}{\partial n} = 0$ the second equation (3) for the function X, which characterizes the vortex deformation of the edge effect due to transverse shear, when determining the critical loads and frequencies of free oscillations, turns out to be unrelated to the other equations of the written system [6].

Because of this, for these problems, the values of this function can be set equal to zero, that is $\chi = 0$ [11]. Under boundary conditions for χ, different from those written out above, the influence of this function on the values of critical loads or oscillation frequencies will affect only when the boundary conditions for the moments are met.

Since the most essential boundary conditions for shells are the conditions imposed on the functions $w, \varphi u \psi$, then it seems natural in the first approximation the assumption that in general $\chi = 0$ and the main system (3) will then consist of three equations for the functions ψ, φ, w.

Let the following conditions be satisfied on the contour for the functions

$$\psi(s) = 0, \frac{\partial^2\psi}{\partial n^2} = 0, w(s) = 0 \,. \quad (4)$$

Integrating equation (1), we obtain:

$$\nabla^2\psi = \frac{\bar{B}_1(1-\nu^2)}{R} w + \Gamma_1. \quad (5)$$

Due to the boundary conditions (4) $\Gamma_1 = 0$.

Substituting (5) into (3), as well as integrating the first equation, we obtain the system of the following equations:

$$D_1\nabla^2\varphi - D_3\nabla^2 w - 2Gh\varphi = 0 \,,$$

$$D_2\nabla^2\nabla^2 w - D_3\nabla^2\nabla^2\varphi + T\nabla^2 w - \lambda^2 w = 0,$$

$$\lambda^2 w = (\Lambda^2 - B_1(1-\nu^2) : R^2). \quad (6)$$

If the boundary conditions for the functions coincide in form (free support, sliding pinching, a combination of free bearing and pinching, etc.), then the solution to system (6) can be sought in the form $\varphi = Aw$.

Taking this into account, equations (6) can be written as

$$\nabla^2 w - \mu w = 0, \nabla^2\nabla^2 w + T_1\nabla^2 w - \lambda_1^2 w = 0, \quad (7)$$

where

$$\mu = A * 2Gh : (AD_1 - D_3), \lambda_1^2 = \lambda^2 : (D_2 - AD_3), \quad T_1 = T : (D_2 - AD_3)$$

(8)

Let us now require that the solution of the second equation in (7) includes the solution of the first one.

We will look for a solution to the second equation (7) in the form:

$\nabla^2 w + \delta w = 0.$ (9)

This will give us two equations:

$$\nabla^2 w + \delta_1 w = 0, \nabla^2 w + \delta_2 w = 0. \quad (10)$$

The condition for the compatibility of solutions to equations (7) is given by the equalities

$$\mu = -\delta_1 \ or \ \mu = -\delta_2, \quad (11)$$

which serve to determine an arbitrary parameter A.

Thus, when condition (11) is satisfied, the solution of the problem of free oscillations of an arbitrary, in terms of a flat spherical shell and a flat plate, will be reduced to solving the well-studied equation for a single-layer spherical shell

$$\nabla^2 \nabla^2 w + T_1 \nabla^2 w - \lambda_1^2 w = 0. \quad (12)$$

When solving specific problems using the proposed method, a difficulty may arise associated with solving the cubic equation for the parameter A, to which condition (11) leads. This difficulty is not fundamental and can be circumvented by the method described below.

As an example, consider the problem of axisymmetric vibrations of gently sloping spherical domes bounded in plan by a circular region.

The corresponding equation (12) at T_1=0 in polar coordinates will be

$$\nabla^2 \nabla^2 w - \lambda_1^2 w = 0. \quad (13)$$

$$\sigma_1 = \lambda_1, \sigma_2 = -\lambda_1$$

Let's consider a sliding pinch. The boundary conditions have the form:

$$w = \frac{dw}{dr} = 0, \qquad \varphi = \frac{d\varphi}{dr} = 0, \qquad \frac{d\psi}{dr} = 0 \quad \text{при} r = a \quad (14)$$

where a -is the radius of the circle bounding the spherical shell.

3. Results and analysis

When the boundary conditions for w are satisfied, due to the relation $\varphi = Aw$ the conditions for φ. Function condition ψ, which takes into account the level of stretching energy, strictly speaking, will not hold. This circumstance, however, will not significantly affect the results, since for a sphere the fulfillment of the boundary conditions is reflected only at the level of the bending energy.

For boundary conditions (14), the smallest oscillation frequency parameter , determined by equation (13), is equal to

$$\lambda_1 = \beta, \qquad \beta = \frac{10.54}{a^2}. \quad (15)$$

Using (8), (11), (13) and (15), we find the parameter $2GhA:(AD_1 - D_3) = -\beta, A = D_3\beta:(D_1\beta + 2Gh)$.

respectively, the oscillation frequency parameter

$$\Lambda^2 = B_1(1-\nu^2):R^2 + \beta^2[\beta(D_1D_2 - D_3^2) + 2GhD_2]:(D_1\beta + 2Gh).$$

This formula exactly coincides with the similar formula in [2]. The calculation results are shown in Fig.1.

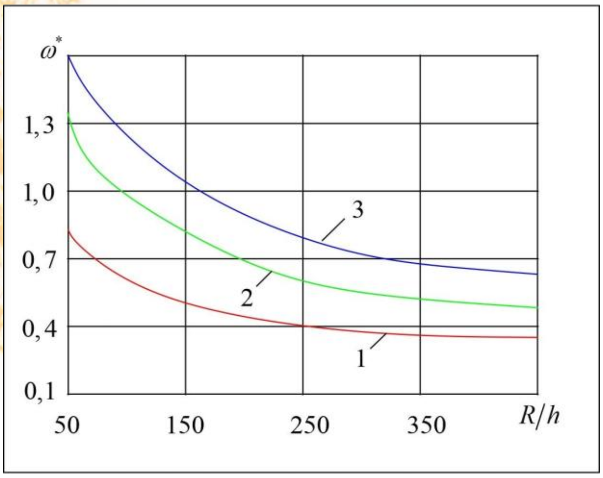

Fig.1. Changing the natural frequencies of the three modes depending on the frequency.

It can be seen from the figures that as the dimensionless parameter R/h increases, the frequency approaches the asymptotics.

4. Conclusion

In conclusion, we note that the problem of natural oscillations of viscoelastic multilayer spherical shells has been posed and solved. An equation of resonant frequencies is obtained, which makes it possible to implement the construction of eigenforms. Note that, without any changes, the method developed here can be extended to the solution of linear problems of the strength of a shallow spherical shell.

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